Implications of Charmless B Decays with Large Direct CP Violation

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Based on the most recent data in charmless B decays including the very recently reported large direct CP violations, it is shown that the weak phase γ can well be extracted without two-fold ambiguity even only from two decay modes $\pi^+\pi^-$ and π^+K^-, and its value is remarkably consistent with the global standard model fit at a compatible accuracy. A fit to all the $\pi\pi, \pi K$ data favor both large electroweak penguin and color-suppressed tree amplitude with large strong phases. It is demonstrated that the inclusion of $SU(3)$ symmetry breaking effects of strong phases and the inelastic rescattering effects can well improve the consistency of the data, while both effects may not be sufficient to arrive at a small electroweak penguin amplitude in the standard model. It is of interest to notice that large or small electroweak penguin amplitude becomes a testable prediction as they lead to significantly different predictions for the direct CP violations for $\pi^0\pi^0$, $\pi^0\bar{K}^0$ modes. Clearly, precise measurements on charmless B decays will provide a window for probing new physics.

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The evidences of direct CP violation in B decays have recently been reported by the BaBar and Belle collaborations. The latest averaged data give $a_{CP}(\pi^+\pi^-) = 0.46 \pm 0.13$ and $a_{CP}(\pi^+K^-) = -0.11 \pm 0.02$[1]. Thus direct CP violation has been established not only in the kaon system, but also in B system. It has been shown that both direct CP violation and $\Delta I = 1/2$ rule in kaon decays can be understood in the standard model (SM)[2]. It is then natural to test whether the observed direct CP violations and decay rates in charmless B decays can be explained within the SM. As the two experimental groups BaBar and Belle have reported more and more accurate data for charmless B decays ($B \to \pi\pi, \pi K$)[3], it then allows one to test the SM and to explore possible indications for new physics, such as the two-Higgs-doublet model with spontaneous CP violation[4], the supersymmetric models etc. There have been several global analyzes which are based on either model independent parameterizations such as quark flavor diagrammatic decomposition [5, 6, 7, 8], isospin decomposition[9], flavor SU(3) symmetry[11], or QCD inspired calculations such as QCD factorization [8, 12] and perturbative QCD approach[13] as well as soft-collinear effective theory[14].

In this letter, we are going to make a step-by-step fit for the charmless B decay modes based on approximate SU(3) flavor symmetry and hierarchical structures of diagrammatic amplitudes. Based on the most recent data including the very recently reported large direct CP violations, we arrive at the following main observations: i) the current data allow us to precisely extract the weak phase γ from only two modes $\pi^+\pi^-$ and π^+K^- without two-fold ambiguity. The resulting numerical value of γ is found to be remarkably consistent with the global SM fit at a compatible accuracy; ii) A direct fit to all $\pi\pi$, πK modes favors a large electroweak penguin. Furthermore, the large or small electroweak penguin amplitude is found to be a testable prediction via measuring direct CP violations in the decay modes $B \to \pi^0\pi^0$ and $\pi^0\bar{K}^0$ once more accurate data become available. iii) all the amplitudes and strong phases in $B \to \pi\pi, \pi K$ are extracted, which indicates large final state interactions and non-factorizable QCD effects as the resulting numerical results show an enhanced color-suppressed tree amplitude and strong phase. It is shown that not only the large $\pi^0\pi$ branching ratios but also the large π^0K^0 ones result in a large color-suppressed tree amplitude with a large strong phase; iv) it is the large π^0K branching ratio that mainly responsible for a large electroweak penguin amplitude with a large strong phase. In the case of a small electroweak penguin amplitude fixed by the isospin relation in the SM, the resulting π^0K branching ratios are bellow the experimental data; vi) SU(3) symmetry breaking of strong phases and $B \to DD$ rescattering effects can well improve the consistency of the global fitting. However, it remains necessary to have a large electroweak penguin amplitude with large strong phases.

The diagrammatic decomposition approach is adopted to carry out a global analysis. The advantage is that in such an approach some decay modes can form, in a good approximation, closed subsets, which allows us to determine the relevant parameters without knowing the others. Although the number of data points decrease for each subset, the number of the free parameters decrease as well. Of interest, the precision of the determinations is not necessarily lower than that of the whole global fit. Furthermore it may avoid the complicity and the potential inconsistency in the current data when more decay modes are involved in the whole global fit. The compar-

ison between different results from different subsets may provide us important hints to understand those decays. In general, all the $B \to \pi\pi$ decay modes can be written in terms of diagrammatic amplitudes: tree ($\mathcal{T}$), color-suppressed tree ($\mathcal{C}$), QCD penguin ($\mathcal{P}$), electro-weak penguin ($\mathcal{P}_{EW}$), color suppressed electroweak penguin ($\mathcal{P}^C_{EW}$) etc. The corresponding diagrams in $B \to \pi K$ are denoted by primed ones, such as $\mathcal{T}'$, $\mathcal{P}'$, etc. Using the CKM factors $\lambda_q^{(s)} = V^*_{qd(s)} V_{qb}$, and the unitarity of the CKM matrix, the penguin type amplitude can be decomposed as : $\mathcal{P}^{(\prime)} = \lambda_u^{(s)}\mathcal{P}_u + \lambda_c^{(s)}\mathcal{P}_c + \lambda_t^{(s)}\mathcal{P}_t$. Defining $P \equiv P_{tc} = \mathcal{P}_t - \mathcal{P}_c$, $P_{tu} \equiv \mathcal{P}_t - \mathcal{P}_u$, $\hat{P}_{EW} = P_{EW} + P^C_{EW}$ and factorize out the CKM factors, we arrive at the following diagrammatic decomposition

$$
\begin{aligned}
\bar{A}_{\pi^+\pi^-} &= \lambda_u(T - P_{tu} - \frac{2}{3}P^C_{EW,tu}) - \lambda_c(P + \frac{2}{3}P^C_{EW}) \\
\bar{A}_{\pi^-\pi^0} &= -\frac{1}{\sqrt{2}}\left[\lambda_u(T + C - \hat{P}_{EW,tu}) - \lambda_c \hat{P}_{EW}\right] \\
\bar{A}_{\pi^0\pi^0} &= \frac{1}{\sqrt{2}}\Big[\lambda_u(-C - P_{tu} + \hat{P}_{EW,tu} - \frac{2}{3}P^C_{EW,tu}) \\
&\quad -\lambda_c(P - \hat{P}_{EW} + \frac{2}{3}P^C_{EW})\Big] \\
\bar{A}_{\pi^+K^-} &= \lambda^s_u(T' - P'_{tu} - \frac{2}{3}P'^C_{EW,tu}) - \lambda^s_c(P' + \frac{2}{3}P'^C_{EW}) \\
\bar{A}_{\pi^0\bar{K}^0} &= \frac{1}{\sqrt{2}}\Big[\lambda^s_u(-C' - P'_{tu} + \hat{P}'_{EW,tu} - \frac{2}{3}P^C_{EW,tu}) \\
&\quad -\lambda^s_c(P' - \hat{P}'_{EW} + \frac{2}{3}P^C_{EW})\Big] \\
\bar{A}_{\pi^-\bar{K}^0} &= \lambda^s_u(P'_{tu} - \frac{1}{3}P'^C_{EW,tu}) + \lambda^s_c(P' - \frac{1}{3}P'^C_{EW}) \\
\bar{A}_{\pi^0K^-} &= -\frac{1}{\sqrt{2}}\Big[\lambda^s_u(T' + C' - P'_{tu} - \hat{P}'_{EW,tu} + \frac{1}{3}P'^C_{EW}) \\
&\quad -\lambda^s_c(P' + \hat{P}'_{EW} - \frac{1}{3}P'^C_{EW})\Big] \qquad (1)
\end{aligned}
$$

where the rescaled amplitudes have a hierarchical structure $T \gg P \gg \hat{P}_{EW}$. The primed and unprimed amplitudes are equal in the SU(3) limit. For simplicity, throughout this paper, we will neglect the smallest amplitudes of P^C_{EW} and take in a good approximation $P_{EW,tu} \simeq P_{EW,tc} = P_{EW}$ and $P_{tu} \simeq P_{tc} = P$ due to t-quark dominance. As a phase convention, we take T to be real, i.e. $\delta_T = 0$. The amplitudes are normalized to the CP averaged branching ratio $Br = (|A|^2 + |\bar{A}|^2)/2$ in units of 10^{-6}, where the tiny differences due to the B^0 and $B^\pm$ lifetime difference and the final state phase spaces are neglected. The direct CP violation is defined through $a_{CP} = (|\bar{A}|^2 - |A|^2)/(|\bar{A}|^2 + |A|^2)$. The flavor SU(3) symmetry breaking effects for amplitudes are considered as $|T'/T| = |P'/P| = |P'_{EW}/P_{EW}| \simeq f_K/f_\pi \simeq 1.28$ from naive factorization. The SU(3) symmetry breaking effects of strong phases are characterized by the phase differences of the primed and the unprimed amplitudes $\Delta\delta_A \equiv \delta'_A - \delta_A$ with A denoting for any of the amplitudes T, P, P_{EW} etc.

The decay modes of $\pi^+\pi^-$ and π^+K^- provide five data points: two CP averaged branching ratios $Br(\pi^+\pi^-) = 4.6 \pm 0.4$ and $Br(\pi^+K^-) = 18.2 \pm 0.9$, two direct CP asymmetries $a_{CP}(\pi^+\pi^-)$ and $a_{CP}(\pi^+K^-)$, and one mixing induced CP asymmetry $S_{\pi\pi} = -0.61 \pm 0.14$. Taking the flavor $SU(3)$ relations and neglecting P^C_{EW}, the two decay modes only involves T, P, δ_P and the weak phase γ. Thus all of them can be directly determined. A fit to the current data gives the following results

$$
\begin{aligned}
&|T| = 0.53 \pm 0.03, \qquad |P| = 0.09 \pm 0.002, \\
&\delta_P = -0.48^{+0.09}_{-0.12}, \qquad \gamma = 1.11^{+0.11}_{-0.14} \qquad (2)
\end{aligned}
$$

with a $\chi^2_{\min}/d.o.f = 0.71/1$. Where the well measured result of $\sin 2\beta = 0.73 \pm 0.037$ from $B \to J/\psi K_S$ has been used to relate the weak phase α to the weak phase γ via unitarity relation. The values of $|T|$ and $|P|$ are well determined with relative errors less than 10%. The error of δ_P is larger but can be reduced with more accurate data in the recent future. The ratio $|P/T|$ is found to be around 0.17. Note that the best fitted angle γ is in a remarkable agreement with the one from the global SM fit of the unitarity triangle which gives $\gamma = 1.08^{+0.17}_{-0.21}$ and at a compatible accuracy. We emphasize that the above results are obtained without the interference with other $\pi\pi, \pi K$ modes in which more diagrammatic parameters C and P_{EW} are involved. Therefore it provides a very promising way to extract γ from charmless B decays and an important reference point for any further analysis. In obtaining the above result, the newly reported $a_{CP}(\pi^+K^-)$ plays a key role. Without it, as shown in ref[5, 6], the determination of γ suffer from a two-fold ambiguity with the other solution at $\gamma \simeq 40°$. In Fig.1, we plot the χ^2_{min} as a function of γ. It is clearly seen that after including $a_{CP}(\pi^+K^-)$ the global minimum (best-fit) of χ^2 falls into the allowed range of the SM fit and the ambiguity is lifted.

Note that in the above fit the $\pi\pi$ and πK modes are related via the SU(3) relations, while the symmetry breaking effects on the strong phases have been neglected. As pointed out in ref. [9] the SU(3) breaking in strong phases may significantly change the correlation between $a_{CP}(\pi^+\pi^-)$ and $a_{CP}(\pi^+K^-)$. Taking $\Delta\delta_P$ as a free parameter in the fit, we find

$$
\begin{aligned}
&|T| = 0.53 \pm 0.03, \qquad |P| = 0.09 \pm 0.002 \\
&\delta_P = -0.67^{+0.24}_{-0.45}, \qquad \Delta\delta_p = 0.21 \pm 0.4, \\
&\gamma = 1.06 \pm 0.2 \qquad (3)
\end{aligned}
$$

with $\chi^2_{min} = 4.7 \times 10^{-7}$, which manifests that a small value of $\Delta\delta_P$ further improves the goodness-of-fit.

When including the branching ratios of $Br(\pi^0\pi^0) = 1.51 \pm 0.28$ and $Br(\pi^0\pi^-) = 5.5 \pm 0.6$ but ignoring P_{EW} at the moment as both modes are dominated by T and C, only two new parameters C and δ_C are involved. A

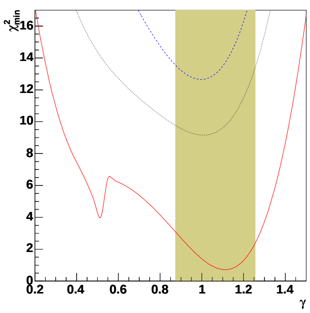

FIG. 1: The χ^2_{min} as functions of γ. The three curves (from bottom to top) are: Solid: Fit to $\pi^+\pi^-$ and π^+K^- data only. Dashed: Fit to all the $\pi\pi$, πK modes, with $\hat{P}_{EW}$ free (Fit B). Dotted: Fit to all the $\pi\pi$, πK modes, with $\hat{P}_{EW}$ fixed by Eq.(5) (Fit A). The shadowed band indicates the allowed range from the global SM fit.

fit to the four decay modes leads to the following results

$$\begin{aligned} &|T| = 0.53^{+0.029}_{-0.03}, |C| = 0.43 \pm 0.05, \delta_C = -0.85^{+0.52}_{-0.28} \\ &|P| = 0.08^{+0.003}_{-0.005}, \delta_P = -0.48^{+0.09}_{-0.11}, \gamma = 1.11^{+0.11}_{-0.14} \end{aligned} \quad (4)$$

which shows a large ratio of $|C/T| = 0.81$. In the QCD factorization estimation this value is bound to be $|C/T| \leq 0.4$. The error of δ_C is significantly large. Note that the values of $|T|$ and $|P|$ and γ remain almost unchanged, which indicates no explicit contradiction between two sets of data $\pi^+\pi^-, \pi^+K^-$ and $\pi^0\pi^0, \pi^0\pi^-$, and the relatively large ratio $|C/T|$ is purely the results of the large $\pi^0\pi^0$ and $\pi^0\pi^-$ branching ratios.

We now include other πK data to determine $\hat{P}_{EW}$ and its strong phase. We use the experimental value of $Br(\pi^0\bar{K}^0) = 11.5 \pm 1.0$, $Br(\pi^-\bar{K}^0) = 24.1 \pm 1.3$ and $Br(\pi^0K^-) = 12.1 \pm 0.8$. The preliminary data of $a_{CP}(\pi^-\bar{K}^0) = 0.02 \pm 0.034$ and $a_{CP}(\pi^0K^-) = 0.04 \pm 0.04$ are also considered. However, we do not include the preliminary data of $a_{CP}(\pi^0\pi^0), a_{CP}(\pi^0\bar{K}^0)$ as we would like to leave them to be pure predictions from the fits. From the isospin structure of the effective weak Hamiltonian in the SM and the relations between the isospin amplitudes and the diagrammatic amplitudes, i.e., $a_2^c = \hat{P}_{EW}$ and $a_2^u = T + C - \hat{P}_{EW}$, one arrives at the following well-known model-independent constraint [10]

$$\frac{\hat{P}_{EW}}{T+C} \simeq -\frac{3(C_9+C_{10})}{2(C_1+C_2)} \simeq (1.25 \pm 0.12) \times 10^{-2} \quad (5)$$

with $C_i s$ being the Wilson coefficients evaluated at m_b. This relation tightly constrains the magnitude and the phase of $\hat{P}_{EW}$. However in the presence of new physics beyond the SM, the ratio could be significantly modified. In view of the recent puzzling experimental results, a careful analysis is urgently needed to find out whether this relation is indeed favored by the data.

We now discuss several cases. First, consider a fit (Fit A) to the $\pi\pi$, πK data using Eq.(5). The result is given in the first column of Tab.I. Comparing with the fit to $\pi^+\pi^-$ and π^+K^- in Eq.(2), one notices that the values of γ, $|T|$ and $|P|$ are almost unchanged. C and its strong phase become larger and the ratio $|C/T|$ is enhanced to be close to ~ 0.9. Namely, the large $\pi^0\pi^0$ branching ratio is actually not the full reason for a large $C/T \approx \mathcal{O}(1)$. It is also required by the πK data. The $\chi^2_{min}/d.o.f$ is found to be 12.7/7 which is much higher than the previous fits. The main inconsistency comes from the branching ratio of π^+K^-, π^0K^0 and $\pi^-\bar{K}^0$. The resulting best-fit values in this case are 20.0 ± 0.8, 9.7 ± 0.5 and 22.3 ± 0.7 respectively. The inconsistencies can be characterized by two ratios $R_n = Br(\pi^+K^-)/Br(\pi^0\bar{K}^0) \simeq 0.79$ and $R = Br(\pi^+K^-)/Br(\pi^-\bar{K}^0) \simeq 0.76$, which should be very close to 1.0 in the SM. The small value of R_n may require corrections to P_{EW} while R may be connected to large non-facotrizable effects [15]. An important feature of this fit is that the predicted direct CP violations $a_{CP}(\pi^0\pi^0) \simeq 0.36$ and $a_{CP}(\pi^0\bar{K}^0) \simeq -0.11$ are large and compatible with $a_{CP}(\pi^+\pi^-)$ and $a_{CP}(\pi^+\bar{K}^-)$. The χ^2_{min} vs γ is also given in Fig.1 which shows a good determination of γ.

	FitA	FitB	FitC	FitD
γ	$1.0^{+0.11}_{-0.13}$	$1.0^{+0.13}_{-0.18}$	$0.98^{+0.12}_{0.13}$	$1.1^{+0.12}_{-0.19}$
$\lvert T\rvert$	0.52 ± 0.27	0.52 ± 0.03	0.52 ± 0.03	$1.13^{+0.36}_{-0.32}$
$\lvert C\rvert$	0.47 ± 0.04	0.45 ± 0.05	0.45 ± 0.05	$0.32^{+0.35}_{-0.22}$
δ_C	$-1.1^{+0.19}_{-0.17}$	$-0.88^{+0.3}_{-0.2}$	$-1.87^{+0.3}_{-0.25}$	$-2.7^{+1.29}_{-0.3}$
$\lvert P\rvert$	0.094 ± 0.001	0.093 ± 0.002	0.09 ± 0.002	0.74 ± 0.3
δ_P	$-0.49^{+0.09}_{-0.10}$	$-0.53^{+0.10}_{-0.14}$	-0.76 ± 0.17	$-0.2^{+0.05}_{-0.14}$
$\lvert\hat{P}_{EW}\rvert$	–	0.03 ± 0.01	0.03 ± 0.01	0.024 ± 0.01
$\delta_{P_{EW}}$	–	$0.67^{+0.2}_{-0.3}$	$0.67^{+0.2}_{-0.4}$	$1.13^{+0.19}_{-0.39}$
$\lvert P_D\rvert$	0(fix)	0(fix)	0(fix)	0.11 ± 0.02
δ_{P_D}	0(fix)	0(fix)	0(fix)	$-0.21^{+0.09}_{-0.14}$
$\Delta\delta_P$	0(fix)	0(fix)	$0.2^{+0.1}_{-0.17}$	0(fix)
χ^2/dof	12.7/7	9.1/5	7.9/4	5.4/3
$a_{\pi^0\pi^0}$	0.36 ± 0.11	0.05 ± 0.22	-0.06 ± 0.2	0.07 ± 0.39
$a_{\pi^0\bar{K}^0}$	-0.10 ± 0.004	-0.01 ± 0.05	-0.02 ± 0.05	-0.01 ± 0.11
$B_{\pi^0\pi^0}$	1.7 ± 0.3	1.56 ± 0.4	1.53 ± 0.4	1.7 ± 0.5
$B_{\pi^0\bar{K}^0}$	9.7 ± 0.48	11.1 ± 1.8	11.1 ± 2.1	11.3 ± 2.3
$B_{\pi^0K^-}$	11.7 ± 0.6	11.9 ± 2.2	11.8 ± 2.4	11.9 ± 2.5
$a_{\pi^+\pi^-}$	0.27 ± 0.06	0.30 ± 0.08	0.37 ± 0.06	0.34 ± 0.27
$a_{\pi^+K^-}$	-0.1 ± 0.02	-0.11 ± 0.02	-0.1 ± 0.03	-0.1 ± 0.06

TABLE I: Best fitted parameters and predictions from charmless B decay data in four different cases. Details are explained in the text.

Second, considering a fit (Fit B) with freeing the parameter $\hat{P}_{EW}$ and its strong phase. The results are tabulated in the second column of table I, which show roughly

the same values of γ, $|T|$, $|P|$ and $|C|$, but the value of $|P_{EW}| \simeq 0.03$ leads to

$$\frac{|\hat{P}_{EW}|}{|T+C|} \simeq (3.1 \pm 1.3) \times 10^{-2} \tag{6}$$

which is twice as large as in Eq.(5). The data of $Br(\pi^0\bar{K}^0)$ and the ratio R_n are perfectly reproduced. This result agrees with the observation in Refs.[6] with a statement that a large electro-weak penguin can consistently explain the πK data. Clearly, the large value $\hat{P}_{EW}$ is driven by the observed large branching ratio of $\pi^0\bar{K}^0$ mode. All the previous fits with small $\hat{P}_{EW}$ failed to meet this date point[7, 8]. Note that in the case of large $\hat{P}_{EW}$, the predicted CP violations of $\pi^0\pi^0$ and $\pi^0\bar{K}^0$ are found to be small. The predicted central values are only 0.06 and -0.02 respectively though the errors are big(see Tab.I). Therefore it provides a possibility to distinguish the electro-weak penguin effects in the near future with more accurate measurements. In the third column (Fit C) of Tab.I, we consider the effects of $SU(3)$ breaking in the strong phases. The best fit gives $\Delta\delta_P = 0.2$ in accordance with Eq.(3). In this case, value of $\hat{P}_{EW}$ remains the same as in Fit.B. The predictions give $a_{CP}(\pi^0\pi^0) = -0.06$ and $a_{CP}(\pi^0\bar{K}^0) = -0.1$ respectively.

The inclusion of all the $\pi\pi$ and πK modes allows one to investigate the possible large inelastic rescattering effect due to the process of $B \to DD_{(s)} \to \pi\pi(\pi K)$. It is well known that $B \to DD$ have a large branching ratio about 50 times greater than that of $B \to \pi\pi$, which amplifies the successive small effects of re-scattering $DD_{(s)} \to \pi\pi(\pi K)$. Considering the fact that $B \to DD_{(s)}$ only contributes to the isospin $0(1/2)$ $\pi\pi(\pi K)$ final sates and carries only the CKM factor $\lambda_c^{(s)}$, its contribution can be parameterized by only one complex quantity denoted by $D(D')$ and effectively it can be considered by replacing $P^{(')}$ by $P_D^{(')} = P^{(')} + D^{(')}$. In the fourth column (Fit D) of Tab.I, the parameters of $|P_D|$ and δ_{P_D} motivated by the inelastic rescattering from $B \to DD_{(s)}$ are added which makes P and P_D two independent parameters. The results show that P_D is compatible with the QCD penguins obtained from the previous fits of A,B,C while P becomes larger. This large difference between P and P_D indicates a large effect of inelastic rescattering and may also imply new physics in strong penguin sector. In this fit, the ratio of $|C/T|$ is reduced to 0.37. The ratio of $\hat{P}_{EW}/(T+C)$ remains large and the two predicted CP violations are again small.

In conclusion, the current data enable us to make a very encouraging global fitting for testing the standard model and probing new physics. It will be very crucial to arrive at more accurate measurements on both branching ratios and direct CP violations in $B \to \pi^0\pi^0$ and $\pi^0\bar{K}^0$. The current preliminary data give $a_{CP}(\pi^0\pi^0) = 0.28 \pm 0.39$ and $a_{CP}(\pi^0\bar{K}^0) = -0.09 \pm 0.14$[1]. Due to the large errors, including them will not change the conclusion. Numerically, we find that the results in Eq.(4) are unchanged. The ratio $|\hat{P}_{EW}/(T+C)|$ remains large and is found to be 0.024 ± 0.01, 0.034 ± 0.01 and 0.033 ± 0.04 for FitB,C and D respectively in Tab.I. It is very likely that we are standing at the corner of finding new physics with two B-factories.

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[1] K.Abe. et.al (Belle) Phys.Rev.**D**012001(2003). B.Aubert, et.al(Babar), hep-ex/0407057. Z.Ligeti, talk at ICHEP04,Beijing,Auguest 16-22(2004),hep-ph/0408267. M. Giorgi, talk at ICHEP04, hep-ex/0408113, Y. Sakai, talk at ICHEP04, hep-ex/0410006.

[2] Y. L. Wu, Phys. Rev. D **64**, 016001 (2001)

[3] For a summary, see: Heavy Flavor Averaging Group, http://www.slac.stanford.edu/xorg/hfag/rare

[4] Y.L. Wu and L. Wolfenstein, Phys.Rev.Lett. **73** 1762 (1994); Phys.Rev.Lett. **73** 2809 (1994); Y.L. Wu and Y.F. Zhou, Phys.Rev. **D61** 096001 (2000).

[5] J. Silva and L. Wolfenstein, Phys.Rev. **D49** 1151 (1994)

[6] A. J. Buras *et al.*, Phys. Rev. Lett. **92**, 101804 (2004). hep-ph/0402112. T. Yoshikawa, Phys. Rev. D **68**, 054023 (2003) D. Atwood and G. Hiller, hep-ph/0307251. S. Mishima and T. Yoshikawa, hep-ph/0408090. S. Nandi and A. Kundu, hep-ph/0407061. D. Atwood and A. Soni, Phys. Rev.**D58**, 036005 (1998).

[7] C. W. Chiang *et al.*, Phys. Rev. D **70**, 034020 (2004). Phys. Rev. D **69**, 034001 (2004).

[8] J. Charles *et al.*, arXiv:hep-ph/0406184.

[9] Y. L. Wu and Y. F. Zhou, Phys. Rev. D **62**, 036007 (2000). Eur. Phys. J. dirC **5**, 014 (2003). Y. F. Zhou, et.al,Phys. Rev. D **63**, 054011 (2001).

[10] M. Gronau, et.al, Phys.Rev.D60:034021(1999). Y.Grossman, M.Neubert and A.L.Kagan, JHEP 9910:029(1999)

[11] X. G. He, et.al, Phys. Rev. D **64**, 034002 (2001)

[12] M. Beneke and M. Neubert, Nucl. Phys. B **675**, 333 (2003) D. s. Du, et.al, Phys. Rev. D **67**, 014023 (2003) R. Aleksan, et.al,Phys. Rev. D **67**, 094019 (2003). G. Buchalla and A. S. Safir, Phys. Rev. Lett. **93**, 021801 (2004).

[13] Y. Y. Keum, H. n. Li and A. I. Sanda, Phys. Lett. B **504**, 6 (2001) Phys. Rev. D **63**, 054008 (2001)

[14] C.W. Bauer et. al., hep-ph/0401188.

[15] A. J. Buras *et al.*, hep-ph/0410407.